Application of Mathematical Modelling: Logistics Network Planning

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Manufacturing companies with commodity products demand constant up-gradation of supply chains to improve their service levels in order to stay relevant in the market and to minimize the risk of losing customers to the competitors. One such specific service level which needs constant adherence and improvement is the "Order Fulfilment Time". Companies are compelled to accept tight deadlines due to the of risk losing business to a competitor providing a better timeline. Accomplishing deliveries within the agreed deadline is challenging and requires an optimized warehouse and distribution network beforehand. This **optimized warehouse and distribution network** not only improves the **customer service level** but also minimizes the **total logistics cost** accruing during transportation of goods. Setting up of small warehouse and distribution network is mostly a qualitative decision taken - based on experience and by employing simple techniques like **clustering**. But in the case of large networks the decision take in quantitative in nature and demands a considerable expertise of **Mathematical optimization** to select an optimal network from a wide set of networks. Though there are many off-the-shelf software packages to optimize the warehouse and distribution network, these packages are more general and limited in their applications especially when minute nuances of the real-world network covering an entire country like India need to be incorporated in the form of decision variables, parameters, constraints, and custom User Interface. Hence, a dedicated team comprising of **Subject Matter Experts, Software Engineers, Data Modelers, Mathematical Optimization Scientists** are required to carry out the task of optimization.

1. Background

This paper derives its motivation from the experience to resolve the challenge of fulfilling the orders within the agreed "**Order fulfilment Time**" at a **Large-Scale Manufacturing** Company producing '**Common Products**'. The finished product is hereby referred to as '**Goods**'. The Cement Production Company hereby referred to as '**Seller'** is in a 'Deliver at Place' agreement with its Dealers/ Traders hereby referred to as '**Buyer**'. According to this agreement, the Seller is obliged to deliver the goods to the Buyer, by bearing all the risks, responsibilities and costs associated with transportation of goods until the buyer's destination. The challenges discussed here arise due to short/ tight "**Order Fulfilment Time**" within which the goods must be transported from the Buyer to the Seller. In the following section, a brief discussion is made on the "Deliver at Place" scenario in which the challenges arise, followed by a sub section discussing a few technical details required to understand the scenario,

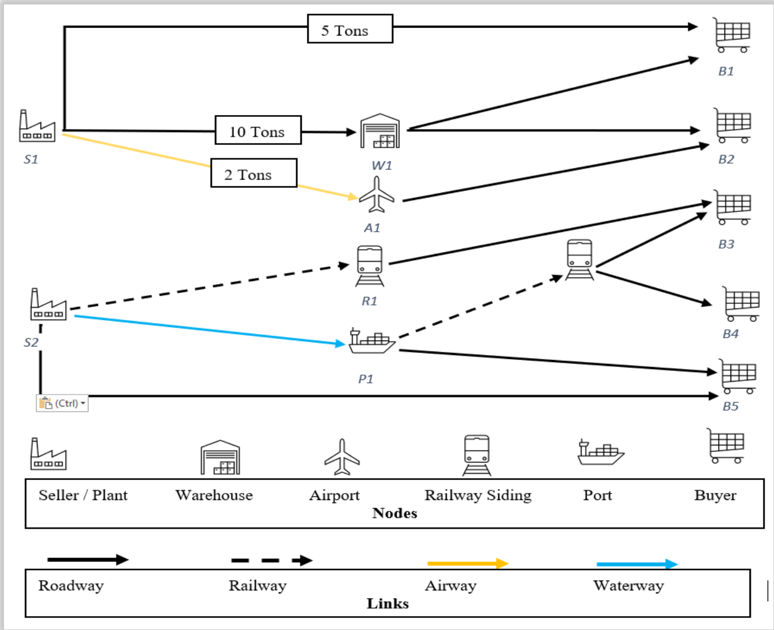
1.1 Why Seller agrees to Deliver at Place? Buyers prefer the "Deliver at Place" agreement because it moves the Cost, Risk and Responsibilities of handling Goods towards the Seller. Even though the entire onus of responsibilities is on the Seller side, the seller is compelled to pick this contract due to,

1. Risk of losing the customer if the Competitors are already providing the 'Deliver at Place' fulfilment.
2. Buyers do not want to do manage the logistics due to the lower Rate of Volume flow.

1.2 Logistics network: To understand the challenges and mitigations arising from tight 'Order Fulfilment Time', a brief understanding of Logistics Network is expected.

Logistics network is schematic representation of '**Warehouse and Distribution Network**' showing the flow of goods from the Seller to the Buyer in the form of a graph **[Exhibit A]**. This graph is made up of two entities Nodes and Links. The Buyer, Seller and Intermediatory points where goods get exchanged such as - Warehouses, Ports, Transits, Railway Sidings – are called the nodes of a network. The Roadways, Railways, Airways, Waterways, Pipelines that connect these Nodes are called links of a network. One common intermediary node is the Warehouse. Often these warehouses are near the Buyer and are replenished as per the requirement. The warehouses are often connected to the Seller (Upstream), Buyer (Downstream) and to other Intermediary Nodes by Roads. Some other intermediary nodes are Railway Siding connected to its upstream through Railway and Downstream nodes through Road. Similarly, there are ports connected to its upstream through Waterways (Sea) and Downstream nodes through Road.

***Exhibit A**: Schematic representation of a Downstream Logistics Network. W_1, A_1, R_1, P_1 are the intermediary nodes where goods make a transit.*



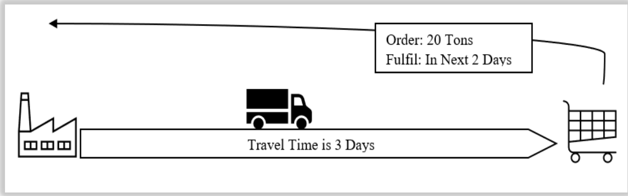
2. Challenges

2.1 Order Fulfilment Time: Order Fulfilment Time is the "The time within which the Seller must deliver the Goods to the Buyer after the latter has placed an order". Failure to meet this deadline leads to lower customer satisfaction or Buyer fulfilling the demand from competitor.

2.2 Challenges arising due to tight Order Fulfilment Time. Often fulfilling the orders within prescribed time is challenging **[Exhibit B]**. Here we list only the **Logistical** challenges due to tight "Order Fulfilment Time",

1. Distance from Seller to Buyer is so large that the transportation time itself exceeds the "Order Fulfilment Time".
2. Availability of Trucks / Logistics at hand for shipment.
3. Small Storage Space at Buyers site forces the Buyer to make small frequent orders. Shipping small orders over large distance is financially expensive. (If the 'Order Fulfilment Time' big enough then enough orders from a region can be clubbed together, thereby increasing the delivery volume).

__Exhibit B__: Challenge of delivering the goods when travel time is greater than the 'Order fulfilment Time.

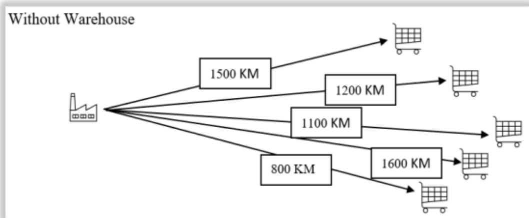


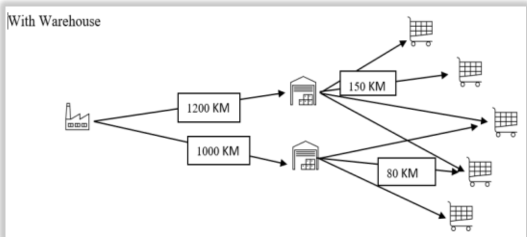
3. Mitigations

3.1 Approaches to mitigate the challenges: In this section we briefly detail the two mitigations to the challenges of delivering the goods within the "Order Fulfilment Time". The mitigations discussed here are advocated by Subject Matter Experts with years of experience in handling Logistics Network. The suggested Mitigations are quantified through the Mathematical Optimization process which are discussed in detail in Section 4.

3.1.1 Warehouse and Inventory: **[Exhibit C]** As discussed, the Seller is not always in the proximity to the Buyer, hence delivery time would often exceed the order fulfilment time. One way to overcome the challenge is to have storage spaces such as Warehouse, Sidings close to the buyer. The advantage of having Warehouses is that the demands on nearby Buyers can be cumulated and forecasted with an increased accuracy such that enough goods are stocked (Inventory) in these storage spaces to suffice the demand for the nearby Buyers before the next replenishment arrives.

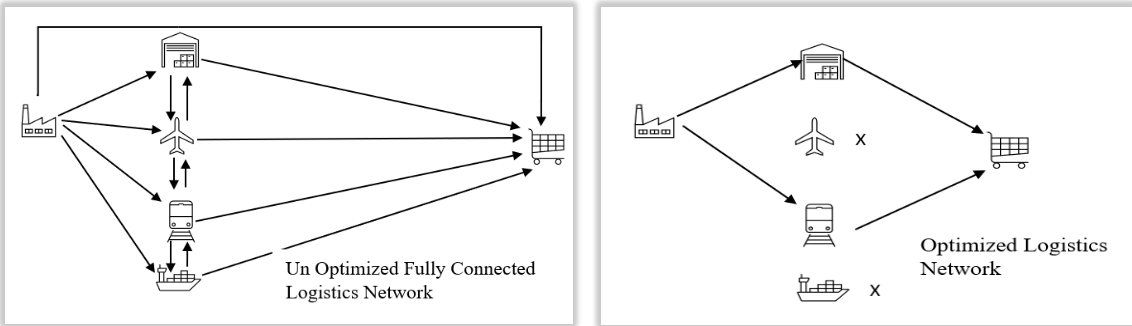
__Exhibit C__: Comparison of logistics network with and without warehouse. In the latter the challenge of longer delivery time is mitigated by placing warehouse in proximity to the Buyers.





3.1.2 Transportation Network: **[Exhibit D]** In addition to setting up the storage space having an Optimized pre-determined Transportation (Logistics Network) along with the mode of transport would eliminate the Last-Minute crunching to arrange the carrier vehicle. In an Un Optimized network each node is connected to other node like a fully connected graph. In an optimized Logistic Network is pruned in such a way that nodes are interconnected with a set of low-cost feasible links. Reduction in number of links will allow the Seller in concentrating on only the set of carriers operating on those links.

Exhibit D*: Comparison of logistics network 'Unoptimized fully connected' vs 'Optimized'. In the latter the logistics team focuses on negotiating the carrier contracts on few optimized routes only.*



4. Mathematical (Integer) Optimization

The two mitigations discussed in the section 3 address the challenge of delivering the goods within "Order Fulfilment Time". In this section we discuss how these mitigations are achieved utilizing the knowledge of Mathematical (Integer) Optimization with the help of a hypothetical illustration. Then a discussion is made on how this illustration can be transformed into real world scenario and the complications in solving these real-world scenarios using off the shelf software.

4.1 What is Mathematical Optimization?

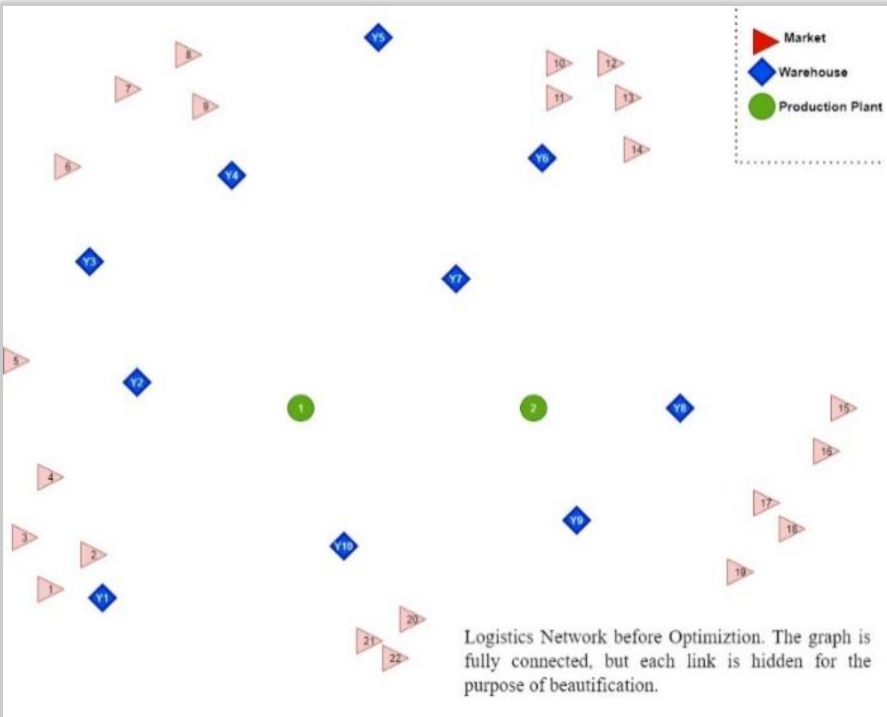
"***Mathematical optimization*** *(alternatively spelled optimisation) or* ***mathematical programming*** *is the selection of a best element, with regard to some criterion, from some set of available alternatives.*". **[Ref 2]**

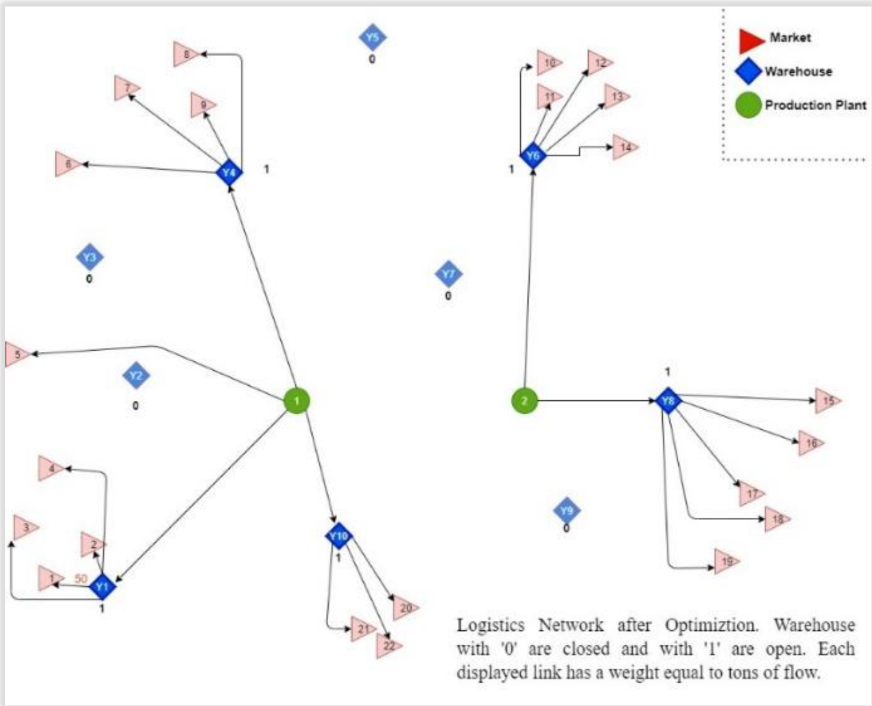
The optimization process used here derives its inspiration from the famous "Facility Location Problem" **[Ref: 3]** of Operations Research, details of which is discussed later in this section. The results of Optimization are employed for decision making at the so called "Tactical Level and Operational Level" of downstream Logistics **[Ref: 4]** as discussed below.

A. At Tactical Level: The process shorts out a small set of intermediary nodes (Warehouses, Railway Siding, Ports where goods make a temporary transit before reaching the seller) from gigantic set of nodes **[Exhibit E]**. The results then are put to making Tactical level decisions such as - Setting up of warehouse, Increasing the capacity of warehouse, negotiating with railways or ports, Negotiating the contracts with carriers between these routes - which usually take few months.

B. At Operational Level: The process prunes the fully connected graph as shown in Fig and outputs optimally connected graph with edges telling the most optimal path to transport the goods **[Exhibit E]**. These results are used in making operational level decision such as – Production targets at the Seller/Manufacturing plant, Inventory to hold at Warehouses, Finalization of the paperwork before dispatching the goods on the path – which usually takes few days.

***Exhibit E**: Comparison of **sample** logistics network before and after Optimization. Optimized network is more concise with clear flow of goods.*





Logistics Network after Optimiztion. Warehouse with '0' are closed and with '1' are open. Each displayed link has a weight equal to tons of flow.

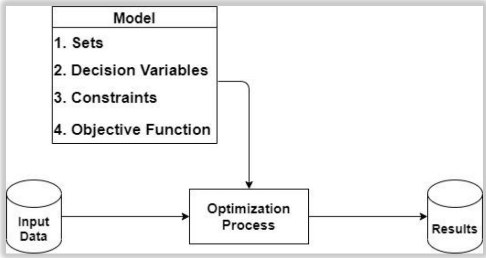
4.2 Facility Location Problem: The optimization process used to achieve the two mitigations discussed in section 3 is built on the principles of a famous problem from Operations Research called the "**Facility Location Problem**" **[Ref: 3]**.

In this section a brief explanation of technical part of the optimization process is done by taking a Sample network as shown in the **[Exhibit E]**. As mentioned earlier, at tactical level the results from the optimization process are utilized in finalizing the intermediary nodes. For sake of simplicity, we consider only one type of intermediary node here i.e., Warehouse.

The sample network has two Sellers/Manufacturers, 10 Potential Warehouse & 22 Buyers. The Seller can either send the goods directly to Buyer or via Warehouse. An assumption is made that the Production Capacity of Seller is greater than the Demand at Buyers.

4.3 The Optimization Process: The entire Optimization process is divided into three parts - A. Input Data Gathering B. Modelling the Scenario C. Output - as shown in the Fig: The intricate details of working of the model are beyond the scope of this paper. **[Exhibit F]**

***Exhibit F**: Three parts of Optimization Process, Input Data, Model and Output.*



4.3.1 Input Data Preparation: In this section we briefly discuss the Input Data Modelling process. This data forms a critical part of Model called parameters.

a. A large set of Warehouses through which the goods can be delivered are identified by Requesting the Information (RFI).
b. Their Latitude and Longitude are marked, and the freight quote is obtained from the Goods Carriers. If the freights between a pair of nodes is not available, then the actual road distance is estimated using tools such as Google Distance API **[Ref: 5]**.
c. This distance is then multiplied by some cost factor to get the freight between two nodes.
d. Other costs/subsidies such as handling cost, Taxes, duties are estimated.

4.3.2 Model: The problem scenario is then modelled using different optimization paradigms, in our case the modelling is done Algebraically a I Integer Programming (IP) Model **[Ref: 6]**. IP model basically has 4 parts as discussed below.

a. Sets: Each unique collection of entities in the model which can be represented by a common blueprint is expressed as a set. The iterations or enumerations are carried out on this set using an index. Such as – The set of Warehouse is represented by 'I' and indexed by 'i'. The set of Buyers is represented by 'J' and index by 'j'.
b. Decision Variables: The decisions to be taken are expressed in the form of variables in the model. In this case the decision to open or close a warehouse is assigned to a binary variable say Y_j (Indexed by 'j') which can take a value of either '0' or '1'. Decision on capacity of a warehouse can be assigned to variable say Z_j (indexed by 'j') which can take values between 'Minimum Throughput Required' to 'Maximum Throughput Allowed".

c. Decision on how much quantity of goods should flow between a warehouse and Buyer is decided by another variable say $X_{j,i}$ which can take values between '0' to 'Maximum flow allowed between two nodes"

d. Constraints: These are the expressions that put some limits on values that decision variables can take Decision Variables. Such as – Limiting the maximum capacity of a warehouse 'j'. Minimum throughput required from a warehouse 'j' to make it active.

e. Objective Function: The direction the optimizer should follow is expressed in the form of objective function. Even though our aim is to mitigate the challenges related to "Order Fulfilment Time", this cannot be done at any random cost without justification. This is where Objective Function plays its role. The Objective function guides the model to achieve the mitigations at the minimum cost or at the maximum profit possible.

4.3.3 Output: The model outputs the results in the form of values to the Decision Variables. Each variable created will have a value of between the limits specified in input data.

As shown in **[Exhibit F]**, the warehouse 1 represented by variable Y_1 has a value of 1 (Open) and warehouse 2 represented by Y_2 has a value of 0 (Close).

Similarly, the link $X_{1,1}$ has a value of 50 which means the model suggests sending 50 Tons of goods over this link.

4.4 Results and Benefits

Primary evaluation of results is done by comparing the KPIs such as "**Total Logistics Cost to fulfil demand for a set of Sellers**" obtained from optimizer to the existing values. After a satisfactory primary evaluation, the optimizer results – potential intermediary nodes to setup and logistics network - are implemented on a cluster of Sellers in small region and monitored for a period, usually two to three quarters to analyse the feasibility of the results in a real-world case. With a successful feasibility analysis, the model along with its results is implemented over the entire serviceable area of the company in a phased manner.

Another application of the Math Model is it serves as '**Turnkey**' tool to analyse different distribution scenarios. The scenarios include,

a. Identification of new Warehouse in a region in case of decommissioning an old Warehouse due to disasters, administrative and maintenance issues.

b. Stress testing the current Warehouse and Distribution (Logistics) Network against a natural calamity.

In such case the optimizer is run only on the existing pruned network with decommissioned Warehouse location removed and potential Warehouse location added to the Math Model (Not on all regions covering entire country as it was done in beginning).

Also, at operational level it was observed that the time to source carrier vehicle was significantly reduced. This observation owes to the fact that optimizer prunes the full connected logistics network to an optimal level, thereby reducing the number of links/routes for which a goods carrier vehicle must be sourced by the logistics management team.

4.5 Modelling Optimization of Real-World Scenarios: The sample presented in section 4.3 is simplistic one. In real world the problems are more complex involving,

A. Different types of nodes such as Railway Sidings, Ship Ports, Airports, E2E Transfer Point.
B. Different types of links such as Railways, Waterways, Pipeline, Airways.
C. Both upstream (Raw Materials) and downstream (Finished Product) logistics.
D. Different tax systems depending on starting and ending node states/region.
E. Flow between nodes is required a SKU level.

Such complexities are difficult to incorporate into the Mathematical model using the 'Off the Shelf Optimization Software' especially which is targeted towards the user with little understanding of Mathematical Optimization. The software is designed to handle more generalized problem rather than handling niche intricate details of model. Also, the User Interface of such software is pretty much standard with little or no room for customization. Hence a necessity arises where in an 'Optimization Application' needs to be built in from scratch to integrate the minute nuances of the real-world scenarios.

4.6 Computational Complexities Optimization of Real-World Scenarios:

As discussed, Real-World scenarios are much more complex in terms of the Mathematical Equations which form the back bones of entire optimization process. This complexity is exacerbated especially when the size of the problem increases exponentially due to large number of 'Nodes' and 'Links' increases which form a part of the problem. The input data and output results comprising such problems are in the order of tens of Gigabytes and require expert level of Software Engineering skills to utilize the limited system resources such as Memory and CPU to solve the problem in reasonable time. Hence in addition to Subject Matter Experts from the Business Domain, Mathematical

Optimization of large-scale real-world scenarios requires expertise from various domains such as - Data Modelling, Software Engineering, Operations Research with knowledge efficiently using professional solvers such as Gourbi, CPLEX, SCIP etc. This mathematical optimization approach not only limited to setting up of 'Warehouse and Logistic Network' but can be extrapolated many other problems of different industries which involves decision making.

5. Conclusion

In this paper a scenario of downstream logistics network consisting of Seller (Manufacturer), Buyers, and Intermediary transits as nodes, with a 'Delivery at Place' obligation on the Seller is presented. From Sellers's point of view an obligation to fulfilling such deliveries within "Order Fulfilment Time" comes with its own challenges. Two approaches to overcome these challenges are suggested by the Logistics Subject Matter Experts i.e. - 'Setting up of Warehouses' and 'Concise and Optimized Logistics Network'. These mitigations are realized in quantitative form by Mathematically Modelling and Optimizing the entire scenario. The output obtained from the Optimization process aids in the decision-making process at Tactical Level and Operational Level of logistics planning. Mathematically modelling the scenario is a complex process which requires expertise from wide range of domains. This methodology Mathematical Optimization can be applied to various types of industries to achieve warehousing and logistical solution.

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